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Doctoral Thesis Abstract

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Abstract

An important and active research line in Non-commutative Algebra is the line known as "Commutative Theorems". The final aim in it is to find conditions that guarantee the commutativity of a ring.
This thesis can be placed in this area of work and conditions studied are related to several types of generalizations of derivations. In a concrete way, conditions have been applied to prime or semiprime rings.

So in Chapter 1 the notions of l-generalized and r-generalized reverse derivations have been introduced. These notions extend the one of reverse derivation. In particular we have proved that the existence of such a map (l-generalized or r-generalized reverse derivation) in a semiprime ring implies the existence of a non-zero central ideal. Furthermore, if the ring is 2-torsion free, then the notions of l-generalized, r-generalized reverse derivations, l-generalized and r-generalized derivations coincide.

In Chapter 2 we have considered maps on a semiprime ring that are left or right generalized derivations on a Lie ideal U. In particular we proved that if (F, d) is a l- and r-generalized derivation and $F^2(U) = (0)$ then $d(U) = F(U) = (0)$ and $d(R), F(R) \subseteq C_R(U)$. On the other side, if (F, d) and (G, g) are right and left generalized derivations, respectively, and $F(u)v = uG(v)$ for all $u, v \in U$, then $d(U), g(U) \subseteq C_R(U)$.

In Chapter 3 center-like elements have been studied. Different generalized centers have been defined and they have been proved to be all equal to the center of the ring R when R is prime.

In Chapter 4 the notion of orthogonality has been considered. We have studied, in particular, the existence of a l-generalized and a r-generalized derivation that are orthogonal, finding necessary and sufficient conditions for their existence. We have got some consequence of orthogonality on the composition of the maps.

In Chapter 5 we have studied rings having a symmetric generalized biderivation that satisfies

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some algebraic conditions on elements of a Jordan ideal J. When the considered ring R is prime and the biderivation is non-zero then we could conclude that $J \subseteq Z(R)$.

Finally, in Chapter 6 multiplicative (generalized)-derivations of semiprime rings have been considered. We have studied some algebraic conditions to derive consequences on the multiplication in the ring.

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